DOCUMENT RESUME

ED 306 124 SE 050 529

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TITLE The Flagpole Factory: Providing a Referent for Linear Equations.
PUB DATE 89
NOTE 17p.; Paper presented to the International Association for Computers in Education (New Orleans, LA, April 1988).
PUB TYPE Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Computer Assisted Instruction; Computer Oriented Programs; *Computer Software; Computer Uses in Education; *Equations (Mathematics); Mathematics Education; *Mathematics Instruction; Mathematics Tests; *Problem Solving; Secondary Education; Secondary School Mathematics; *Word Problems (Mathematics)
IDENTIFIERS *Computer Assisted Problem Solving; Mathematics Education Research

ABSTRACT

A change of emphasis in mathematics education may be needed from a mechanical and computational focus to a problem-solving approach. A first step in facilitating this goal is to place children in situations emphasizing problem-solving skills, requiring them to develop and apply their own logic structures. This paper describes a microcomputer based interactive icon processor for use in helping students develop a referent base for the solution of systems of simultaneous linear equations. The instructional model used in this paper proceeds from the initial use of manipulatives through abstraction via four transitional problem types: manipulatives, sketches, mental pictures, and abstraction. The processor as an anthropomorphic expert is available when help or feedback is needed, and as currently implemented, possesses a certain degree of responsiveness to individual differences in problem-solving strategy. In addition, the computer keeps a chronological record of the operations a student using the software performs, as well as the number, content, and sequencing of hints given. A sample word problem and solution steps are appended. (YP)

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During the past decade society has come to place ever increasing demands upon its members to utilize information in problem solving. As costs of information technologies have decreased, increasingly sophisticated tools have come within the reach of larger and larger sections of the population. These new tools are capable of data storage, recall, and manipulation far beyond ordinary human capabilities. Such tools are not, however, capable of independently solving problems based upon this tremendous mass of data. They are data-laden, but unable to make the linkages between this data and the real world. Their information must be interpreted, filtered, and applied to be of use in solving problems posed in the world. These tasks currently must be performed by human intervention. To function in this setting people must be adaptable and capable problem solvers. Yet, we find many people in the same position as the machines which provide them with data.

The view is becoming accepted that consequential knowledge does not include rote memorization. As Sternberg (1984a, 1984b) puts it, consequential knowledge involves deciding what information is important to learn and incorporating that information into the already existing knowledge base. Yet, in looking at the elementary educational experiences of students in mathematics Davis (1974), Erlwanger (1973), and more recently Peck, Jencks, and Connell (in press) have found that the focus has been upon memorizing facts and rules, not on making sense of the subject. Instead of developing skill in making use of information in a meaningful way, students spend large amounts of time merely

processing and sorting collections of data, the very tasks which computer technology does so well. Quite often, what little problem solving is offered consists of problem specific rules, with little instruction in generalizable strategies for manipulating information in goal-oriented situations. Students so trained come to view mathematics as a system of rules to be memorized and retrieved. In their minds successful mathematical thinking in the classroom becomes either rote recitation of tables or case specific utilization of rules, memorized facts, or miscellaneous data on command. Lacking the ability to make decisions based on their own judgment, verifying the correctness of an answer or a process is left to a source outside of themselves.

Peck, Jencks and Connell (1985) have argued that a primary cause for these difficulties in elementary mathematics lies in the application of a rote memorization teaching methodology in which students routinely are required to memorize procedures in isolation, sidestepping the development of a referent base. Keil (1984) points out that humans are capable of engaging in complex chains of problem solving when it is embedded within and done in reference to a specific knowledge structure or referent base. However, when conceptual referents are not present for the mathematical symbols being manipulated, Schoenfeld (1983) and others suggest that people construct undesirable models concerning knowledge and their role in acquiring it, models which block the development of unifying structures for the information they possess. For example, problems are viewed as always having unique, specific answers, which are wholly determined not by the logic of the problem but by the answer book, a neighbor, or the teacher. As a result, problems in mathematics are approached from unproductive viewpoints, with greater emphasis placed upon recalling memorized rules than in analyzing the situation to be evaluated.

What is needed then is a change of emphasis in mathematics education. Taking a clue from Keil (1984), structuring knowledge with respect to real world referents should play a substantially more important role than the rote mastery of arbitrary rules governing process. When students possess such a reality base they are able to recombine features into new, successful relations in the course of problem solving. Without such a mapping from the abstract symbols to the real world, it is difficult to apply even elementary mathematical meta-cognitive techniques (Campione, Brown, and Connell, in press). Lacking concrete referents, students are unable to identify when the problem situation causes misapplication of their developed rules.

In spite of the mechanical, computational focus of traditional mathematics curricula, there emerge groups of children who seem to naturally organize their thinking in ways that are conducive to problem solving. Kachuk (April, 1987), Connell (April, 1988), and others report that the thought processes and structuring strategies these students utilize are markedly different from other students in the same classroom settings, even though their peers may be considered equally capable in other respects. Students who are good problem solvers possess many linkages relating the subject matter to elements of their real world experiences. These linkages provide a referent base, allowing them to assume ownership over their work and to readily address questions such as "How can you tell?" or "What would happen if ...?" in regard to their final answer, the process by which the answer was obtained, or the underlying premises upon which the process was based.

Unfortunately, the ability to attack and solve problems often appears to have developed independently of school experiences. Evidence suggests that many educational experiences in traditional settings contribute to the formation of barriers which inhibit further conceptual growth. As Spiro, Vispoel, Schmitz,

Samarapungavan, and Boerger (in press) discuss, problems develop as students routinely memorize facts without opportunity to relate these facts to the real world and its intricacies. As discussed earlier, this is particularly problematic in elementary mathematics. Klahr (1984) points out that children often have the ability to produce and manipulate symbols well before they understand what the symbols represent. To the child, such symbols bear no relationship to recognizable facets of the world; hence, the child fails to perceive the underlying ideas and concepts. Nonetheless, a demonstrated ability in the production and manipulation of symbols, however arbitrary or nonsensical these symbols may appear to the student, may lead to instruction proceeding before the student has grasped the concepts that the symbols represent. Given these perceptions and strategies, it is little surprise that students do not value problem solving skills and do not learn them well. They never acquire the conceptual building blocks with which to link their memorized data into meaningful structures.

In order for this situation to be corrected a substantially different curriculum base, presentation schemata, system of psychological rewards, and setting of instruction must be provided. A first step in facilitating this goal is to place children in situations emphasizing problem solving skills, requiring them to develop and apply their own logic structures. A legitimate problem in this setting would involve working on concepts that are within reach given currently possessed knowledge structures. The problem must be new, but within conceptual grasp and existing analytic powers. Problem solving strategies involving posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error should be developed and utilized to develop a referential base for later application. These problem

solving activities should take place at a concrete level, with problems designed to achieve curriculum goals using elements familiar to the child.

The instructional model utilized in this paper was adapted by one of us, Connell (1986), from Robert Wirtz's (1979) model of mathematical problem solving. It proceeds from the initial use of manipulatives through abstraction via four transitional problem types. For the purposes of discussion we shall refer to these problem types as:

1) *Manipulatives*
2) *Sketches*
3) *Mental Pictures*
4) *Abstraction*

An example of physical manipulatives in this model might be a pile of pebbles used to illustrate elementary addition. A sketch would then be drawn recording the actual pile of pebbles. A mental picture is an internal representation of the external sketch. Abstraction occurs when addition is no longer described in terms of countable piles of pebbles, but in terms of pure number.

1) Manipulatives. The power of a physical manipulative lies in the structures which can be built upon it, the linkages it enables in the mind of the student, and its power in explaining concepts. The merit of a manipulative is that it can be used to simplify information, generate new propositions, and increase the manipulability of a body of knowledge.

In thinking of manipulatives it is important to remember that all problems have their origins in the real world about us. The symbolism adopted derives as a result of formal attempts to solve those problems. Although there is certainly a single correct answer for the majority of problems, Hogben (1983) points out that much of what we accept as the correct method for solving a specific

problem has resulted from accidents of notation which have little to do with the underlying logic or mathematics of the situation. By focusing upon the logic that lies beneath the rules, however, we can expand the role of conscious control significantly.

2) Sketches. The sketches follow the form of the original manipulatives as closely as possible. Ideally, the mapping from manipulative to sketch, sketch to mental picture, and finally mental picture to abstraction should be as smooth as possible. If we select an appropriate manipulative, the subsequent sketch draws much of its descriptive power from it.

3) Mental Pictures. In developing a mental picture the student must internalize the informational structure encoded in the sketch. At this time there are many conflicting theories concerning the mechanisms behind the creation and utilization of mental imagery as reflected in the work of Cooper & Shephard (1984), Sawyer (1964), Jencks & Peck (1972) Tweney (1987) and others. They agree, however, that whatever is going on in the brain when we have an image produces a representation that has certain useful functional properties in structuring and organizing information. In applying this model, one must exercise care lest familiarity with a sketch be confused with possession of the underlying mental representation. A sketch is based upon one instantiation of a specific problem type; a mental representation corresponds to a more generalized and broadly applicable knowledge structure.

4) Abstraction. The final step lies in the mental structuring into a more abstract and formal setting. The student has completed the sequence of internalizing the real world problem into justifiable processes by which it may be solved. This setting can then be used in future problems, and as a stepping stone towards independent investigations. If we are successful in following the steps outlined in this model the student will possess not just a single answer

schema, but an entire structural linkage which can be utilized by the student in varied circumstances. The student has developed a sound conceptual building block which can be used in later, more complex endeavors in problem solving.

It is in helping students make the leap from the sketch to the abstraction that the power and potential of the microcomputer can play a much greater role than is currently utilized. The tremendous flexibility of the microcomputer makes it possible to create learning environments utilizing the very presentation schemata, system of rewards, and instructional settings hinted at earlier. In an effort to address this need, we are developing a microcomputer based interactive icon processor for use in helping students developing a referent base for the solution of systems of simultaneous linear equations. The icon processor displays a well developed and flexible mental representation in the form of user-controllable graphics objects (icons) which are in turn based upon plausible physical manipulatives. This program, written in the IBM Handy authoring language (1), is implemented on a 640K IBM PC AT with an EGA graphics card. Our initial efforts have concentrated upon creating a flagpole world within which problems involving two equations and two unknowns may be addressed and solved (2).

In this flagpole world, flagpoles are constructed graphically on the computer screen using various numbers of labeled long and short flagpole sections, corresponding to variables in formally presented algebraic equations. In using the program, the student is initially presented with a graphic representation showing the lengths of two distinct flagpoles formed from integer combinations of long and short sections. These initial flagpoles always correspond to a consistent system of two equations, each of which has two unknowns. For the student, the final goal is to use the icon processor to derive

the lengths of the long and short sections. In working towards the solution, various operations are available to the student for manipulating the flagpoles themselves. For instance, flagpoles may be made longer by integral coefficients, corresponding to the elementary row operation of multiplying a single equation by a constant; flagpoles can be compared and the difference computed, corresponding to the elementary row operation of subtracting one equation from another; and so on. At each stage of this process the newly created flagpoles are displayed graphically together with their associated values, if known, and this information is available for further use by the student. Comparing strategically constructed flagpoles leads to derivation of the lengths of the component sections, equivalent to solving the system for each unknown. In the Appendix we present a sample problem worked through using the graphic representation system developed for "The Flagpole Factory" software.

Such a world, conceived as consisting of dynamically changing configurations of graphics objects, each of which has associated properties, is ideally implemented on a microcomputer. The icon processor developed thus far is capable of manipulating flagpoles according to the user's directions, although more is planned for it in the future. This program has the potential to lead the learner to understand concepts underlying linear algebraic algorithms. For example, imagine the situation in which the student uses a multiple of one flagpole which when subtracted from another flagpole results in the removal of all sections of a certain type. When this is done we have, in a formal sense, accomplished the elimination of one of the variables. Once the length of the remaining section type is known, this newly discovered value can then be used to determine the length of the other section. Because the work is done from within a graphical representation, all of this is done quite intuitively by using the program to compare developed icons and to perform potentially interesting

manipulations of these quantitative objects. When we view this sequence of operations from a formal perspective, however, we see that the student is building a flexible referent for later formal concepts such as Gaussian elimination and back-substitution.

Of course, for a computer-assisted instructional program to be of use in a drill and practice situation, intelligible feedback must be provided by the program to the student. The position we have taken is that for instructional purposes, we must provide more than a black-box which always gives the right answer; the box itself must be transparent and its methods obvious and analogous to those ultimately desired of the student interacting with the computer. This has required the development of an expert capable of using thought processes similar to those found in a skilled user. This anthropomorphic expert is then available to be called upon to act as an advisor when help or feedback is needed, and as currently implemented, possesses a certain degree of responsiveness to individual differences in problem solving strategy. Even in simple two by two systems of equations, multiple solution paths are possible. Either variable may be solved for first, although given the configuration of the problem, it may be more economical, in terms of the number of operations to be performed, to choose one variable over the other. Although there may be a unique optimal solution, what is more important to reinforce is the general strategy by which any system may be solved. The expert has thus been coded not to force only one solution path upon the student. If the student is determined to solve for the long section first and asks the expert for help, the next step in that solution path will be provided, even if it might be easier in the particular system to solve for the short section first. However, if the student has no idea as to how to proceed, the expert will choose to present the easiest solution path (i.e., the one requiring the fewest steps). Furthermore, if the

student has developed from the original problem facts any flagpole which might lead him or her closer to the solution, the expert will make use of that flagpole in suggesting how to proceed, rather than constructing a new flagpole which might lie on a different solution path and possibly confuse the student.

Besides providing expert feedback, the program can play other important roles in the educational process. The computer keeps a chronological record of the operations a student using the software performs, as well as the number, content, and sequencing of hints given. This trace can illustrate differences in the approach utilized during successful and unsuccessful attempts at problem solving. Such information can be invaluable in determining the domain knowledge, heuristics, and the control strategies utilized by the students which, as argued by Collins, Brown and Newman (in press), is critical in the design of effective learning environments.

Acknowledgments

(1) Handy is an experimental language for writing interactive educational software currently under development at the IBM T. J. Watson Research Center, Yorktown Heights, NY. We wish to acknowledge the cooperation Don Nix and Brad McCormick, Handy's designer and implementer, respectively, in providing us with a test version of the language to use in our research.

(2) Original inspiration for the flagpole world came from an adaptation done by Donald M. Peck and Stanley M. Jencks of W. Warwick Sawyer's (1964) Man and Sons problem

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Appendix

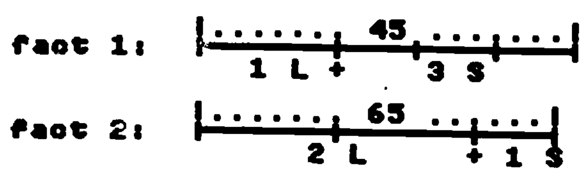
Visually-Oriented Solution of a 2 x 2 System of Linear Equations

Sample Word Problem

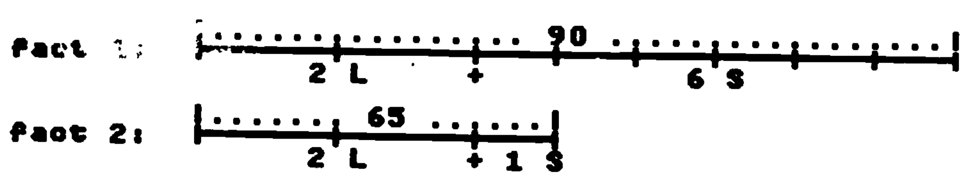
A flagpole factory has two different machines used to manufacture flagpole sections. These machines can be set to make any length of section, but once set they cannot be changed until the next day. On Monday there were two different lengths of flagpole sections, one longer than the other. The length of 3 of the shorter section and 1 of the longer is 45 feet. The length of 2 of the longer and 1 of the shorter is 65 feet. What are the lengths of each type of flagpole section?

Solution Steps

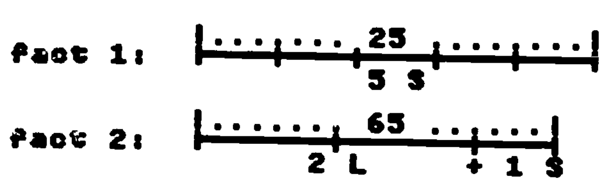
(1) Draw the flagpoles to represent the problem facts.



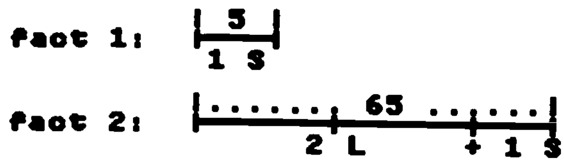
(2) Make fact 1 twice as long to get a common number of long sections (this corresponds to the elementary row operation of multiplying an equation by a non-zero constant).



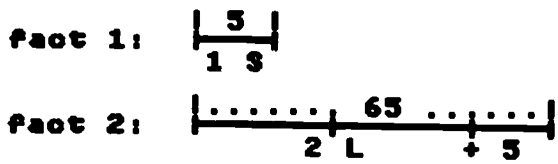
(3) Remove fact 2 from fact 1 (this corresponds to subtracting one row in the matrix from another).



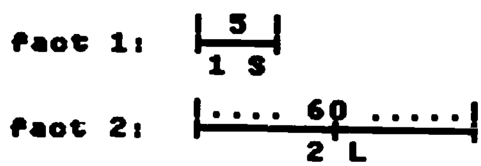
(4) Divide fact 1 by 5.



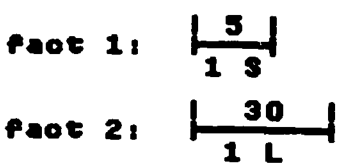
(5) Enter the length now determined for the short section into the other flagpole (i.e., backsubstitute the value of the known variable).



(6) Remove the length of the known flagpole section from fact 2 (i.e., eliminate the effect of the S variable).



(7) Divide fact 2 by 2.



(8) The length of each type of flagpole section (i.e., the value of each variable) is now known. Long sections (L) are 30 feet long; short sections (S) are 5 feet long.